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and if, further, this inequality does not arise from imperfections in Airy's tables which do not exist in Hansen's tables, then all these values will be affected by this inequality. Introducing then all these elements of uncertainty, the results can be written in the form

$$\text{Airy} \quad (P) = -124{\cdot}53 + \frac{1}{3}B,$$

$$\text{Stone} \quad (P) = -124{\cdot}55 - \frac{1}{7}B - \frac{1}{9}\delta V - \frac{1}{7}\delta M,$$

$$\text{Newcomb} \quad (P) = -124{\cdot}36 + \frac{2}{5}B - \frac{1}{9}\delta V - \frac{1}{7}\delta M.$$

These results are those which, we believe, really represent the true values of the apparent coefficient of the parallactic inequality to be derived from the investigation of these astronomers when properly corrected. A comparison of these results with those originally given will show the importance of the correction which we have been obliged to introduce, and the necessity which existed for a rigid examination of the theoretical basis of their labours.

[The concluding portion of this Paper, containing a new discussion of the Greenwich Meridian observations of the Moon in the years 1862–1876, will be published in the June No.—Ed.]

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## *Note on the Astronomer Royal's Investigation of the Theoretical Value of the Acceleration of the Moon's Mean Motion.* By Professor J. C. Adams, M.A., &c.

I lose no time in pointing out briefly the reason why the Astronomer Royal, in the investigation which he communicated to the Society at the last Meeting, has failed to find my value of the coefficient of the Lunar Acceleration.

It may be useful, in the first place, to recall to mind that, according to my theory, the secular changes of

$n$, the Moon's mean motion,

and $e'$, the eccentricity of the Earth's orbit,

are connected by the following relation :—

$$\frac{dn}{ndt} = \frac{e'de'}{dt}\left\{-3m^2 + \frac{3771}{32}m^4 + \frac{34047}{42}m^5 + \ldots\right\},$$

where $m$ denotes, as usual, the ratio of the Sun's mean motion to that of the Moon.

If we stop at the first term of the series within the brackets the result is identical with that found by Laplace.

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We do not know why Laplace did not carry his investigations further than this first term; but he probably thought that the succeeding terms would prove to be inconsiderable.

It is seen, however, that these terms have very large numerical coefficients and that their sign is contrary to that of the first term, and on calculation it is found that the sum of the series is less than its first term nearly in the ratio of 3 to 5.

Hence the secular acceleration will be diminished in the same ratio, and its amount in a century, instead of being about 10″, will be reduced to nearly 6″.

No investigation of the Moon's secular acceleration can be satisfactory which does not take into account terms of the nature of those which give rise to the terms involving $m^4$, $m^5$, &c., above referred to.

There is nothing to object to in the general principles of the method adopted by the Astronomer Royal, but in the practical application of the method I notice very grave defects.

In the first place, the only periodic terms which are included in the Astronomer Royal's expressions for $T\frac{r}{a}$ and $P\frac{r}{a}$ and for the factors multiplying

$$\delta\frac{a}{r},\quad \frac{d}{dt}\left(\delta\frac{a}{r}\right),\quad \delta v,\quad \frac{d}{dt}(\delta v),\quad \&c.,$$

on the right-hand side of the equations, are those which involve the angle 2D or F; whereas it will be seen by a reference to my paper in the *Philosophical Transactions* for 1853, that a great part of the coefficient of $m^4$ in the value of $\frac{dn}{ndt}$ there obtained arises from the combination of terms involving the angles S, F−S and F+S in the expressions for the Moon's coordinates with similar terms in

$$\delta\left(\frac{a}{r}\right),\quad \delta v,\quad \&c.$$

In the present investigation terms of the forms last mentioned are simply ignored.

In the next place, it is to be noted that, although periodic terms depending on the angle F are introduced into the assumed values of $\delta\frac{a}{r}$ and $\delta v$, yet in art. 12, the value of $h$ which is the coefficient of $t^2$ in the value of $\delta v$, is found equal to $-Bb$, quite independently of the values of the coefficients $e, f, g, k$ and $l$, which occur in the terms thus introduced.

The result of this is to reduce the secular acceleration practically to its first term only; which accounts for the coincidence of the Astronomer Royal's value with that of Laplace.

It may also be remarked in reference to art. 11, that although

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terms involving the argument 2F or 4D may be properly omitted, we must put

$$\sin^2 F = \frac{1}{2} - \frac{1}{2}\cos 2F,$$

$$\text{and } \cos^2 F = \frac{1}{2} + \frac{1}{2}\cos 2F,$$

and the constant terms in these latter quantities should be taken into account.

After these general remarks, we will enter a little more closely on the consideration of one or two points in the investigation which are important.

Adopting the Astronomer Royal's notation, let

$\sigma$ denote the Sun's mass,
A the semiaxis major of the Sun's (or Earth's) orbit,
E the eccentricity of the orbit,
R the radius vector at any time.

Then it may be shown, as in the paper before us, that the mean value of

$$\frac{\sigma}{R^3} \text{ is } \frac{\sigma}{A^3}\,\frac{1}{(1-E^2)^{\frac{3}{2}}}, = \frac{\sigma}{A^3}\left(1+\frac{3}{2}E^2\right) \text{ nearly.}$$

Hence if E receive the variation $\delta E$ in the time $t$, this quantity will be increased in the ratio of $1+3E\,\delta E$ to 1 nearly, or in the ratio of $1+bt$ to 1, calling

$$3\frac{E\delta E}{t} = b.$$

Having arrived at this point, the Astronomer Royal assumes that the variation of the disturbing forces due to the variation $\delta E$ in the eccentricity of the Sun's orbit will be represented by supposing

$$\text{T to be replaced by } T(1+bt),$$

$$\text{and similarly P to be replaced by } P(1+bt),$$

and therefore that the new forces, the effects of which are to be found by the present method, are $Tbt$ and $Pbt$ respectively.

On consideration, however, it will appear that this is only true for the non-periodic term in P, and that the periodic terms, whether in P or T, will be changed by any given variation of E in very different ratios.

For instance, the periodic terms in both T and P which depend on the angle 2D or F will vary nearly in the same ratio as $1-\frac{5}{2}E^2$ does, instead of in the ratio in which $1+\frac{3}{2}E^2$ varies as in the above case.

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Hence these terms will be changed by the above-mentioned variation of E in the ratio of $1 + b't$ to 1, where

$$b' = -5\frac{E\delta E}{t} \text{ nearly.}$$

Again, the periodic terms in T and P which depend on the angles S, F−S and F+S will vary nearly in the same ratio as E does, so that these terms will be changed in the ratio of $1 + b''t$ to 1, where

$$b'' = \frac{\delta E}{Et} \text{ nearly.}$$

Hence we see that the values of $b'$ and $b''$ are quite different from that of $b$ which belongs to the non-periodic term, and that $b''$ is much larger than the other two quantities.

The correct way of finding δT and δP, the changes of the disturbing forces T and P due to change in the eccentricity of the Sun's orbit, is to express T and P in terms of the Moon's coordinates $v$ and $r$, the Sun's mean longitude L and its mean anomaly S, and the eccentricity E.

Hence δT and δP may be at once expressed in terms of $\delta v$, $\delta r$, and δE.

Thus calling V the Sun's longitude, and employing the other symbols in the sense before explained, we have

$$P = \frac{1}{2}\frac{\sigma r}{R^3} + \frac{3}{2}\frac{\sigma r}{R^3}\cos(2\nu - 2V),$$

$$T = \qquad -\frac{3}{2}\frac{\sigma r}{R^3}\sin(2\nu - 2V).$$

Or,

$$Pr = \frac{1}{2}\frac{\sigma r^2}{R^3} + \frac{3}{2}\frac{\sigma r^2}{R^3}\cos(2\nu - 2V),$$

$$Tr = \qquad -\frac{3}{2}\frac{\sigma r^2}{R^3}\sin(2\nu - 2V).$$

Now, by the formulæ of elliptic motion, we may find

$$\frac{1}{R^3} = \frac{1}{A^3}\left[1 + \frac{3}{2}E^2 + 3E\cos S\right],$$

$$\frac{1}{R^3}\cos(2\nu - 2V) =$$

$$\frac{1}{A^3}\left\{\left(1 - \frac{5}{2}E^2\right)\cos(2\nu - 2L) + \frac{7}{2}E\cos(2\nu - 2L - S) - \frac{1}{2}E\cos(2\nu - 2L + S)\right\},$$

$$\frac{1}{R^3}\sin(2\nu - 2V) =$$

$$\frac{1}{A^3}\left\{\left(1 - \frac{5}{2}E^2\right)\sin(2\nu - 2L) + \frac{7}{2}E\sin(2\nu - 2L - S) - \frac{1}{2}E\sin(2\nu - 2L + S)\right\},$$

neglecting terms involving 2S, and powers of E above the second.

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Substituting, and then taking the variation, we have

$$\delta(\mathrm{P}r) = \frac{\sigma}{\mathrm{R}^3} r\delta r + 3\frac{\sigma}{\mathrm{R}^3} r\delta r \cos(2\nu - 2\mathrm{V}) - 3\frac{\sigma}{\mathrm{R}^3} r^2\delta\nu \sin(2\nu - 2\mathrm{V})$$

$$+\frac{1}{2}\frac{\sigma r^2}{\mathrm{A}^3}\Big[3\mathrm{E}\delta\mathrm{E} + 3\delta\mathrm{E}\cos\mathrm{S}\Big]$$

$$+\frac{3}{2}\frac{\sigma r^2}{\mathrm{A}^3}\Big[-5\mathrm{E}\delta\mathrm{E}\cos(2\nu - 2\mathrm{L}) - \frac{7}{2}\delta\mathrm{E}\cos(2\nu - 2\mathrm{L} - \mathrm{S})$$

$$-\frac{1}{2}\delta\mathrm{E}\cos(2\nu - 2\mathrm{L} + \mathrm{S})\Big]$$

$$\delta(\mathrm{T}r) = \quad - 3\frac{\sigma}{\mathrm{R}^3} r\delta r \sin(2\nu - 2\mathrm{V}) - 3\frac{\sigma}{\mathrm{R}^3} r^2\delta\nu \cos(2\nu - 2\mathrm{V})$$

$$-\frac{3}{2}\frac{\sigma r^2}{\mathrm{A}^3}\Big[-5\mathrm{E}\,\delta\mathrm{E}\sin(2\nu - 2\mathrm{L}) + \frac{7}{2}\delta\mathrm{E}\sin(2\nu - 2\mathrm{L} - \mathrm{S})$$

$$-\frac{1}{2}\delta\mathrm{E}\sin(2\nu - 2\mathrm{L} + \mathrm{S})\Big]$$

in which $-r^3\delta\left(\frac{1}{r}\right)$ may be written for $r\delta r$, and the expressions given by the ordinary lunar theory in the case of unvaried eccentricity are to be substituted for $v$ and $r$.

Hence, the expressions for $\delta\left(\mathrm{T}\frac{r}{a}\right)$ and $\delta\left(\mathrm{P}\frac{r}{a}\right)$, which are employed in the paper, are wholly incorrect, except in the case of the non-periodic term, which gives rise to the principal term of the secular acceleration or that found by Laplace.

The remark made near the close of the paper, viz. that the magnitudes of the quantities A, B, C, and therefore also that of the secular acceleration are proportional to the inverse cube of the Sun's distance, or to the cube of the Sun's parallax, can only be the result of inadvertence, as the Astronomer Royal himself will be the first to acknowledge.

In fact, the quantities A, B, C involve the factor $\frac{\sigma}{\mathrm{A}^3}$ and this is equal to $n'^2$, where $n'$ is the Sun's mean motion and is known. The Sun's mass $\sigma$ is determined by means of the parallax from this equation; or conversely, if the Sun's mass be known the parallax is thereby determined.

The values of A, B, C are approximately as follows

$$\mathrm{A} = \frac{3}{2}m^2, \quad \mathrm{B} = \frac{1}{2}m^2, \quad \mathrm{C} = \frac{3}{2}m^2,$$

where $m$ denotes, as before, the ratio of the Sun's mean motion to that of the Moon.

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Errata in *Monthly Notices*, vol. xix. p. 207, l. 6, for $-3m^4$ read $-3m^2$.

p. 208, l. 15 for $\frac{dn}{n^2dt}$ read $\frac{dn}{ndt}$.